University of BRISTOL

FOUNTAIN CODING WITH DECODER SIDE INFORMATION

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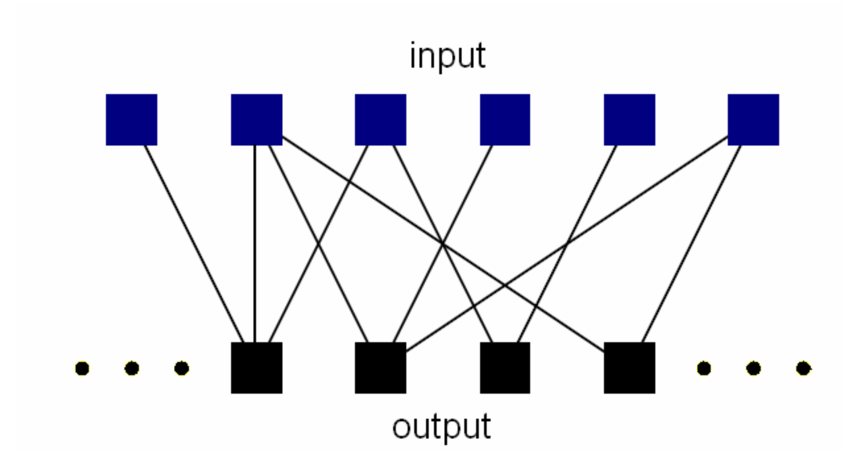
Outline

- Review of fountain codes
- Distributed joint source channel coding (DJSCC) multicast: a priori side information about source at each receiver
- Source-channel fountain code design
  - Analytic relationship of code parameters for good channel and good source-channel fountain codes
  - Optimization of code parameters
- Reduction of the problem with systematic Raptor design
- Simulation results

Fountain codes: TX



transmitter produces a potentially infinite number of fountain encoding symbols (random, equally important descriptions of the source) and sprays them across the channel

[Byers, Luby, Mitzenmacher, Rege 1998]

Fountain codes: RX

receiver collects encoding symbols and attempts decoding when "enough" symbols are received

k : size of information sequence

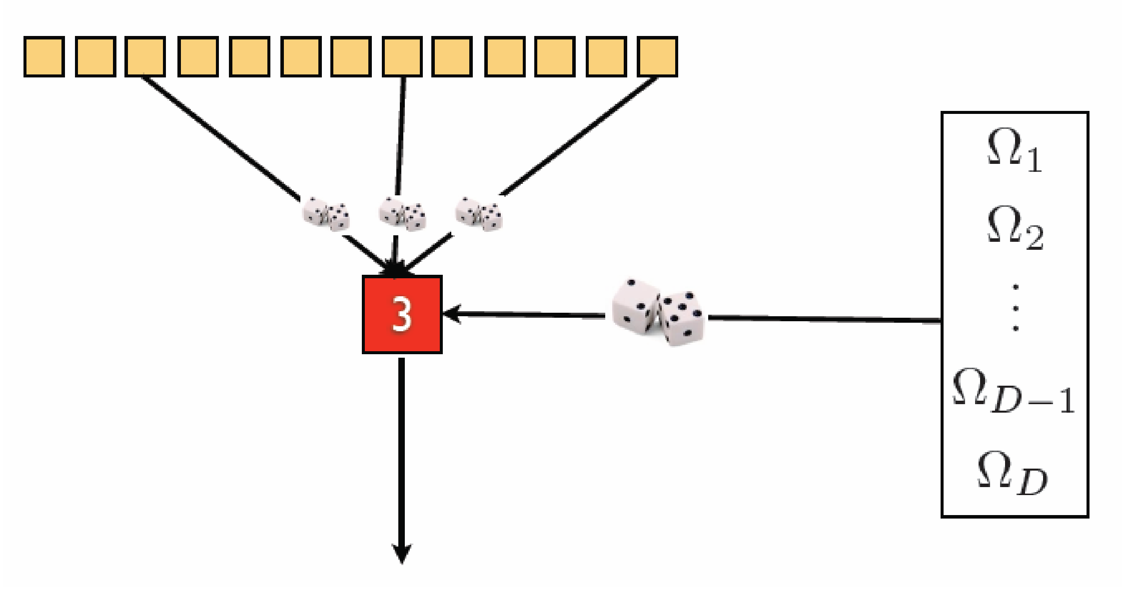
k` : "enough"

erasure channel: k` should be only slightly larger than *k*

noisy channel: k` should be only slightly larger than *k* / Capacity

Sparse graph fountain codes

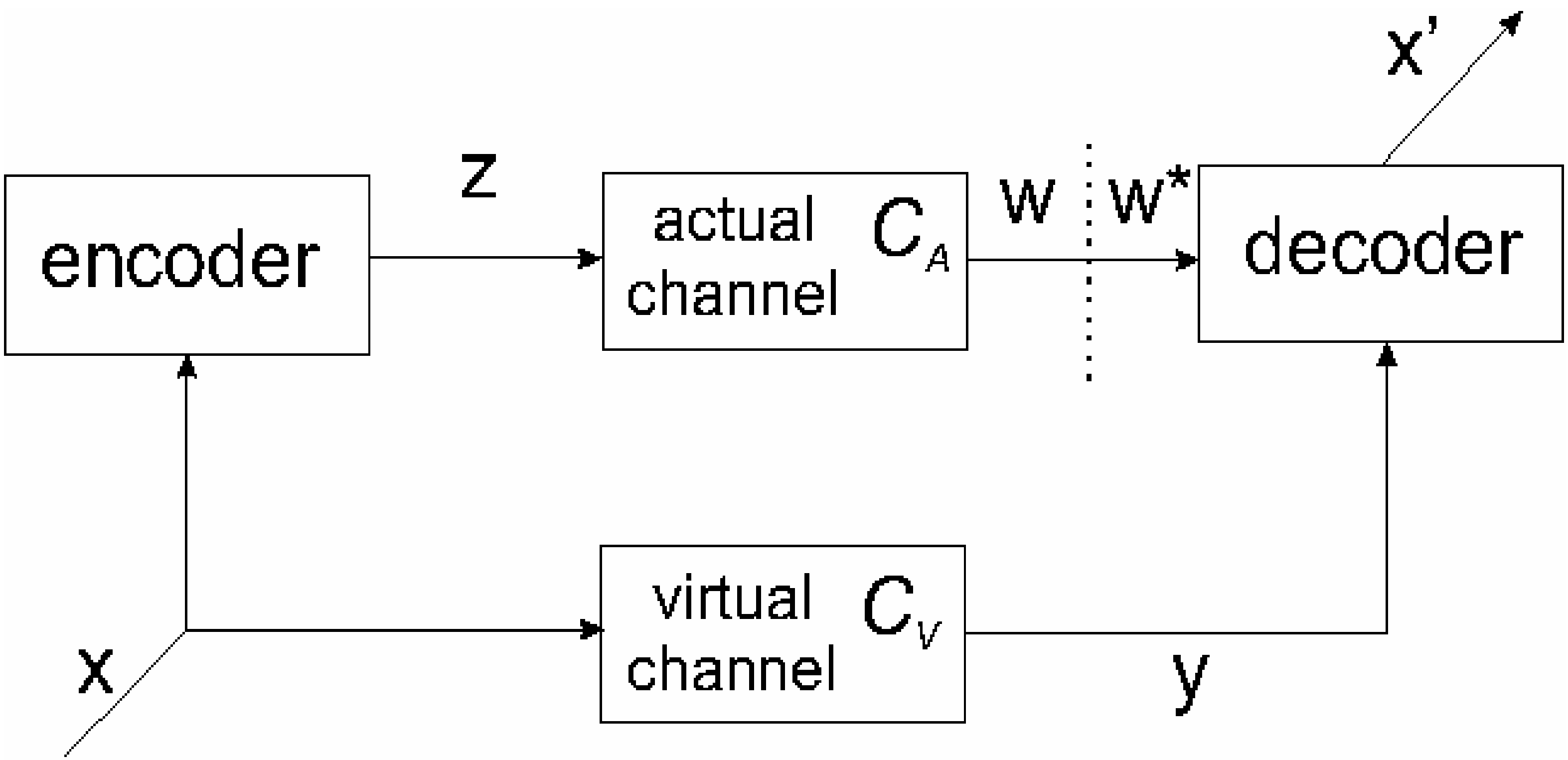
- LT codes [Luby, 1998-2002]
- Raptor codes [Shokrollahi, 2003-2006]



Assumptions

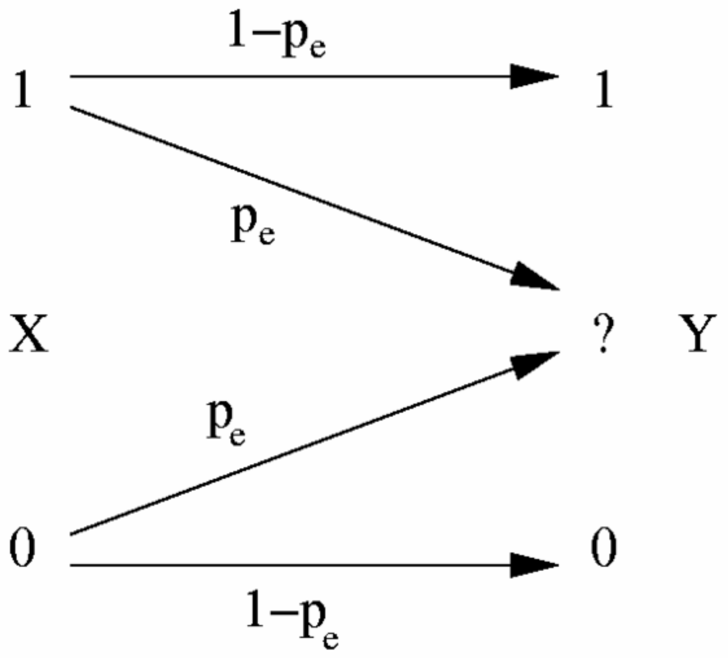
- Each receiver has access to some side information **Y** correlated with **X**.
- We model side information as if **Y** was the output of some virtual communication channel when **X** (uncoded) is its input.
- A single fountain code for both source compression and error correction near the optimal source-channel code rate of H(**X**|**Y**)/Capacity(C).

System model

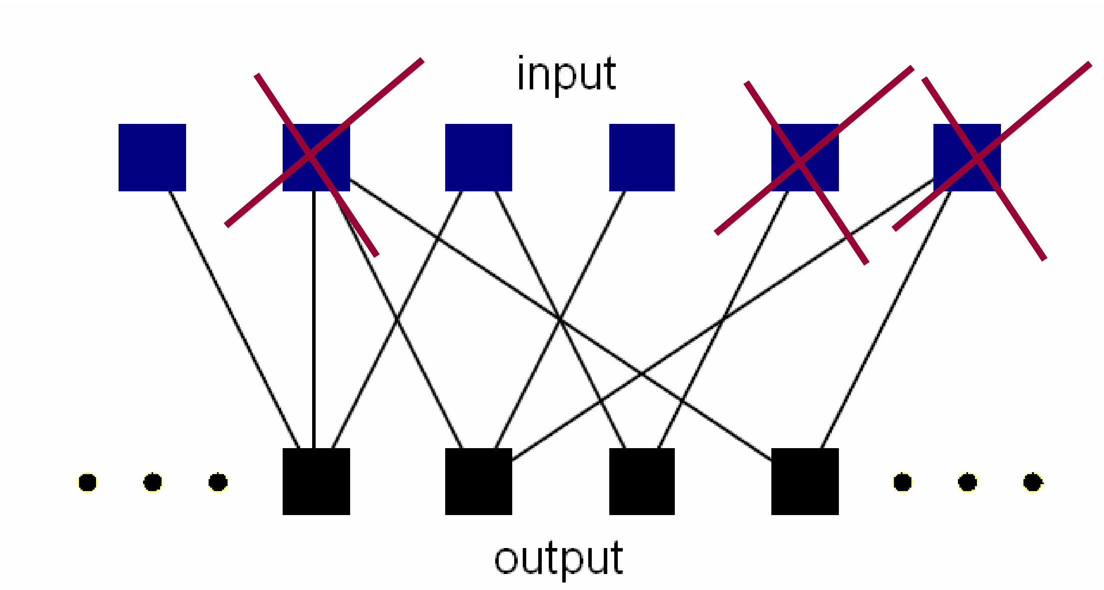


Case 1: Virtual channel as the BEC

- Each receiver has knowledge of *some portion* of the information sequence.
- The transmitter does not know *which* part of the data is available at the receiver, but is able to estimate *how much* of the data is already known.



Using side information in decoding



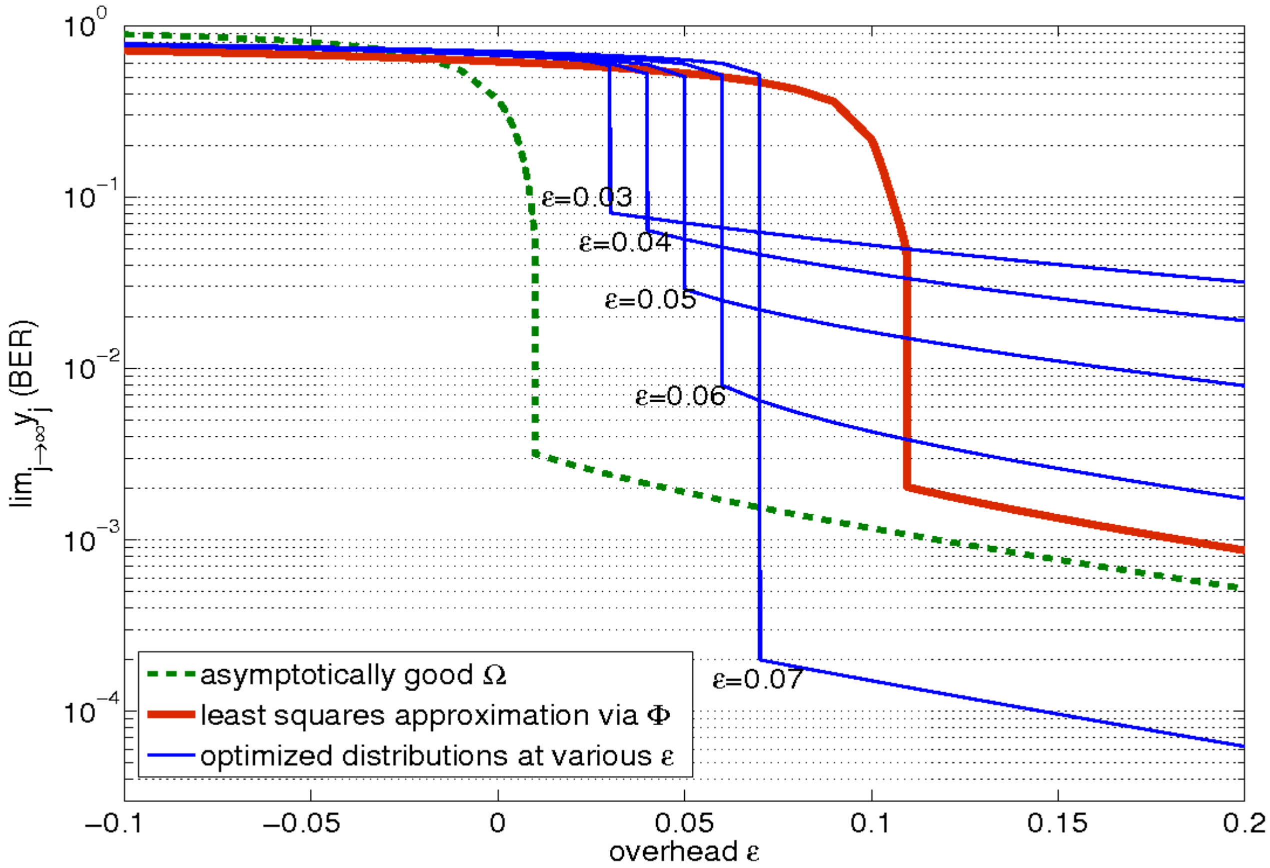
Removal of the known input nodes from the graph changes the degree distribution at the output nodes!

$$\Omega(x) = \Phi(1 - p + px)$$

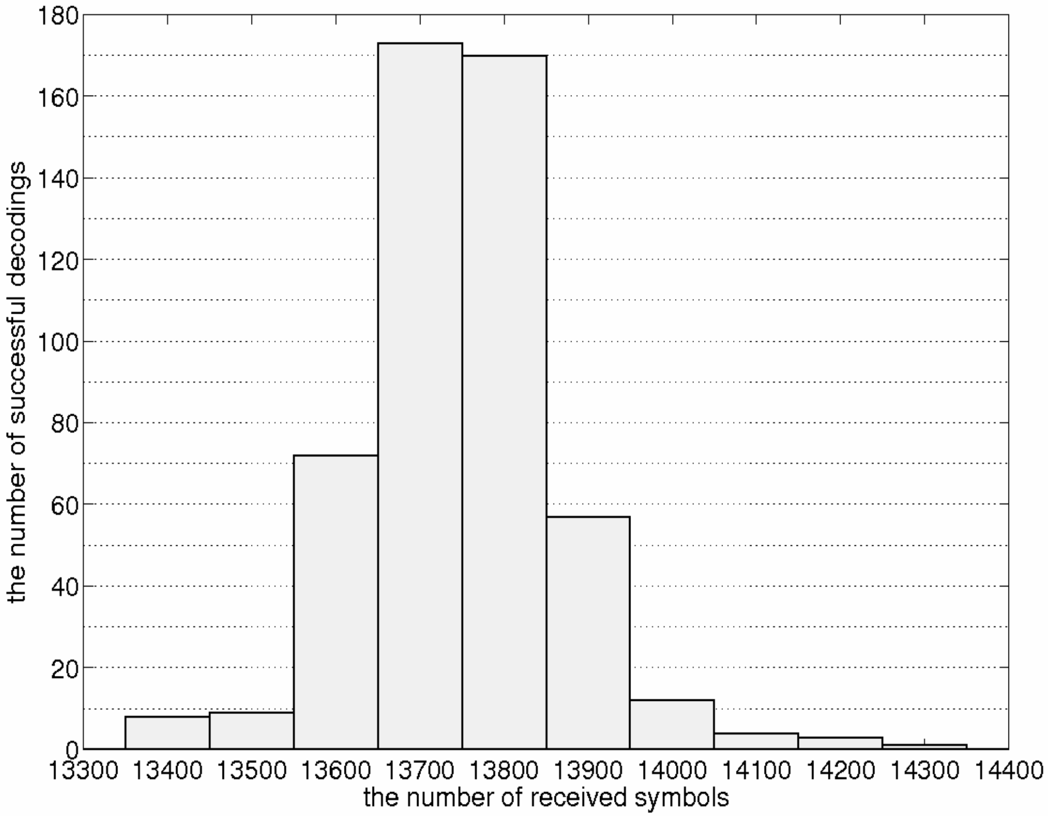
Using relationship between incoming and resulting distribution

- The design goal is to choose such incoming distribution Φ so that the resulting distribution Ω behaves like a good fountain output symbol degree distribution.
- Numerical methods - nonnegative least squares approximation.
- Linear programming optimization of Φ using density evolution of asymptotic BER.

Asymptotic performance of the optimized dd's



Precode + source-channel LT code



k = 40000, p = 0.3, actual channel: BEC

34% of the length of information sequence on average suffices

Related work

- S. Agarwal, A. Hagedorn, A. Trachtenberg, “Rateless codes under partial information”, *Information Theory and Applications Workshop,* UCSD, San Diego, USA, 2008

Applications: data synchronization scenarios

Case 2: Virtual channel as the BIAWGNC

- **Y=X+N**, **N=**$\mathcal{N}(0, \sigma^2)$
- The transmitter does not have access to **Y**, but is able to estimate the noise variance of the virtual channel, i.e., the correlation between **X** and **Y**.
- The receiver embeds the apriori "soft information" into the factor graph prior to running sum-product algorithm.

$$m_{v,f}^{(i)} = \begin{cases} L(y_v) & , i = 0, \\ L(y_v) + \sum_{g \neq f} \mu_{g,v}^{(i-1)} & , i \geq 1. \end{cases}$$

$$\tanh\left(\frac{\mu_{f,v}^{(i)}}{2}\right) = \tanh\left(\frac{L(z_f)}{2}\right) \prod_{u \neq v} \tanh\left(\frac{m_{u,f}^{(i)}}{2}\right), \ i \geq 0$$

Optimization of dd for Case 2

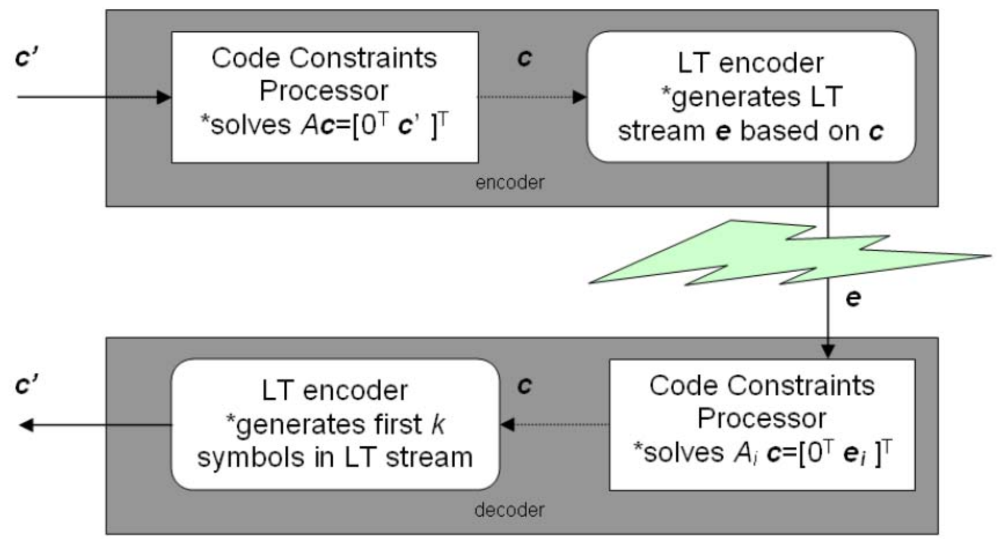
- Modification of linear programming optimization of fountain degree distributions [Etesami, Shokrollahi 2006]
- Semi-Gaussian approximation of density evolution [Ardakani, Kschischang 2004]

another way to resolve fountain decoding with DSI: *systematic raptor*

$G_R = G_{LT} G_C$ is overall Raptor encoding, G_{LT} - truncated LT matrix matrix. Set G_k equal first k rows of G_R.

Idea: if G_k is invertible let $\widehat{x} = G_k^{-1} x$ be the new input to be processed by standard Raptor.

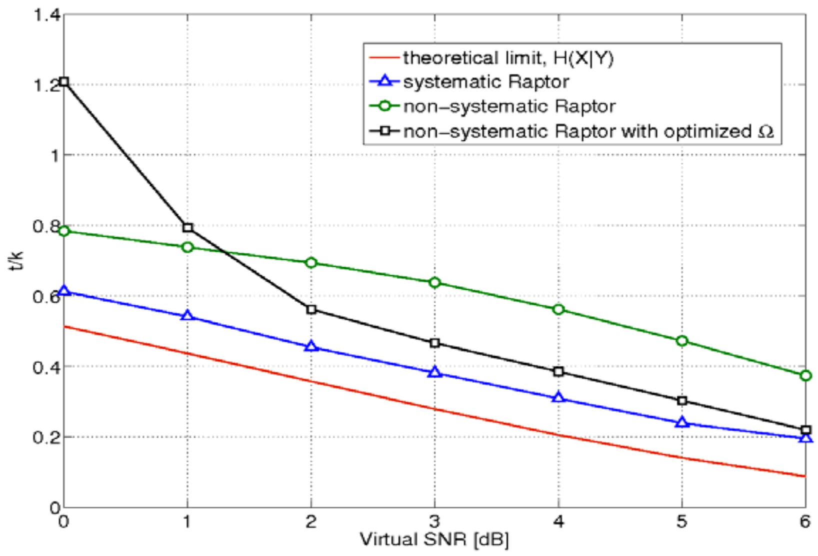
First k symbols of the output stream y will be the same as the input!



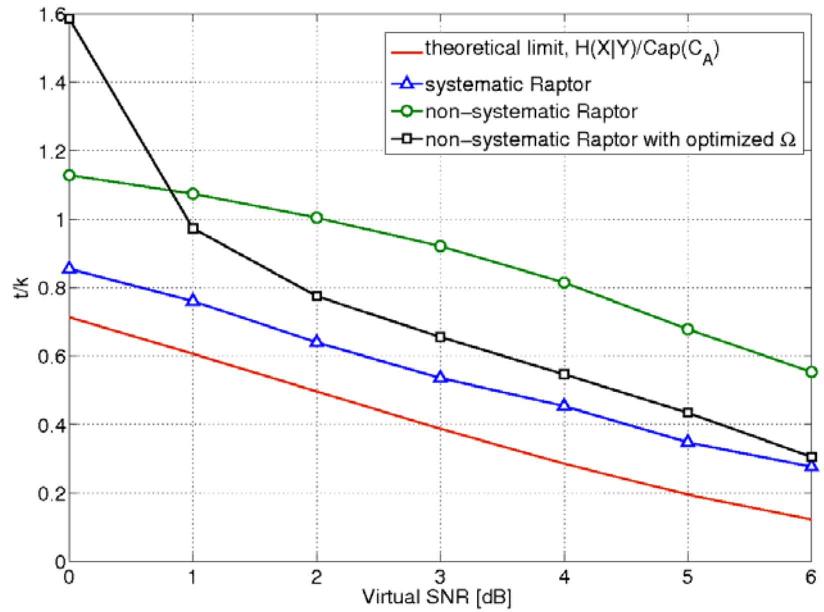
this kind of system is being adopted for application-layer FEC for 3GPP MBMS, IP-datacast for DVB-h…

Reduction to channel coding

- The transmitter multicasts only the non-systematic Raptor encoding symbols.
- The receiver views the available side information as the output corresponding to the systematic symbols.
- No code design modifications are necessary.
- The price of higher encoding/decoding cost, Gaussian elimination at the transmitter for each message block.



k~3000, noiseless actual channel



k~3000, BIAWGN actual channel with 3 dB SNR

Conclusion

- Study of LT and raptor code design when some side information about source is available at the receivers
- Optimization procedure for output symbol degree distributions for non-systematic LT and raptor codes for different models of side information
- Application of systematic raptor design to fountain coding with decoder side information
- Implicit optimal systematic LT design [Nguyen, Hanzo 2008]

Thank you!